

Force and Laws of Motion

Balanced and Unbalanced Force

Balanced and Unbalanced Forces: An Overview

Have you ever played tug-of-war? The rope seems to come to rest momentarily even when so large forces are acting on it from opposite ends. The rope moves occasionally in either direction. Ultimately, the group that pulls harder and applies a larger force wins the game. If you are wondering about the cause of the momentary rest and the occasional motion of the rope in either direction, then you will get the answers in this lesson.



In daily life, we perform different types of activities such as opening a drawer, hammering a nail, moving a table etc. Whether it is to stop a moving object or to move a stationary one, an effort or force is required to perform the activity. In this lesson, we will learn more about the concept of force.

Force and Its Effect

Force is a push or a pull. It tends to change the **motion** of an object. When you push a table, the force makes the table move; consequently, the table is no longer in its state of **rest**.



Effects of Force

1. Force can stop a moving body.
2. Force can move a stationary body.
3. Force can change the speed or direction or both of a moving body.
4. Force can change the shape and size of a body.

Fundamental Forces in Nature

Physics has revealed that all the forces occurring in different contexts arise from a few fundamental forces in nature. These are as follows:

- Gravitational forces
- Weak nuclear forces
- Electromagnetic forces
- Strong nuclear forces

Gravitational Forces

- These were proposed by Isaac Newton.
- These are forces of mutual attraction between masses. The gravitational force between masses is directly proportional to the product of the masses, and inversely proportional to the square of the separation between them.
- These are the weakest forces in nature.

Weak Nuclear Forces

- These were discovered during the study of the phenomenon of β -decay in radioactivity.
- These are the forces of interaction existing between elementary particles of short lifetimes.
- These are about 10^{25} times stronger than gravitational forces.

Fundamental Forces in Nature

Electromagnetic Forces

- These are the forces between charged particles.
- The moving charges produce magnetic effects.
- Electric and magnetic effects are inseparable; hence, the term electromagnetic.
- These forces may be attractive or repulsive.
- Electrostatic forces are 10^{36} times stronger than gravitational forces.
- These operate over small distances.

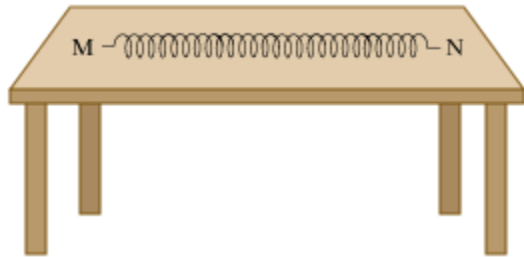
Strong Nuclear Forces

- These forces bind the neutrons and protons together in different nuclei.



- These are short-range forces, operating within the distances of the order of 10^{-14} m.
- These are the strongest forces in nature, being about 10^{38} times stronger than gravitational forces.
- These are attractive forces.

Balanced and Unbalanced Forces on an Object



Suppose a metal spring is placed on a table. Let us say that its two ends are M and N (as shown in the figure). On pulling M, the spring moves toward the left. On pulling N, the spring moves toward the right.

What will happen if you pull M and N simultaneously with the same force?

In this case, the spring will stretch and its shape and size will change; however, it will not move because the net force acting on it is zero.

What will happen if two unequal forces are applied at M and N simultaneously?

When **unbalanced force** is applied at the ends of the spring, it will start moving in the direction of the greater force. In this case, the net force acting on it is not zero.

Did You Know?

Frictional force not only opposes motion but also helps a body to move. It is friction that helps us walk and run.

A toy car is pushed on a rough floor to make it move. The car moves some distance and comes to rest. **Why does it stop moving?**



The toy car comes to rest after some time because of the force of **friction** acting between the moving wheels of the car and the rough floor. This force acts in the direction opposite to the direction of motion of the car. This means that an unbalanced force acts on the car in

the direction opposite to its direction of motion. As a result, it comes to rest after some time. In order to keep the toy car in motion, one should push it again before it comes to rest.

Stopping a Moving Object with an Unbalanced Force

An object moves with a uniform velocity when no net external force is acting on it, i.e., the forces acting on it are balanced.

What will happen if you try to push a table on a rough floor?

The result of the applied force will depend on the magnitude of the force applied on the table.



As shown in the figure, two forces are acting on the table. F_1 is the applied force, while F_2 is the frictional force present between the rough surfaces in contact.

It is clear from the figure that the frictional force opposes the applied force. The table will not move if the applied force is lesser than the frictional force. On applying a force greater than the frictional force, the table will move in the direction of the applied force.

Did You Know?

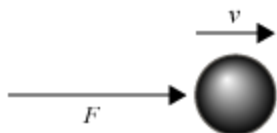
The earth is acted upon by the gravitational pull of the stars, planets, sun and moon. However, the gravitational pull of the sun is the most dominant.

Unbalanced Force and Acceleration/Deceleration

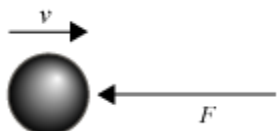
An unbalanced force can change either the speed or the direction of motion of a moving body. As a result, the body can acquire positive or negative acceleration. Hence, an unbalanced force is required to accelerate a uniformly moving body. The direction of acceleration depends on the direction of the unbalanced force.

Case I: Suppose an unbalanced force F is acting on a uniformly moving ball in the direction of its motion. In this case, the ball will accelerate in its direction of motion.





Case II: Now suppose the unbalanced force F is acting on the uniformly moving ball in the direction opposite to its direction of motion. In this case, the ball will decelerate and come to rest.



The speed (v) of the ball will increase and decrease in 'Case I' and 'Case II' respectively. If the force F is removed, then the ball will move with a uniform speed. This is because no net force will be acting on it.

When you stop pedaling a bicycle, it starts slowing down. Name the forces that act on the bicycle. Also mention their directions.

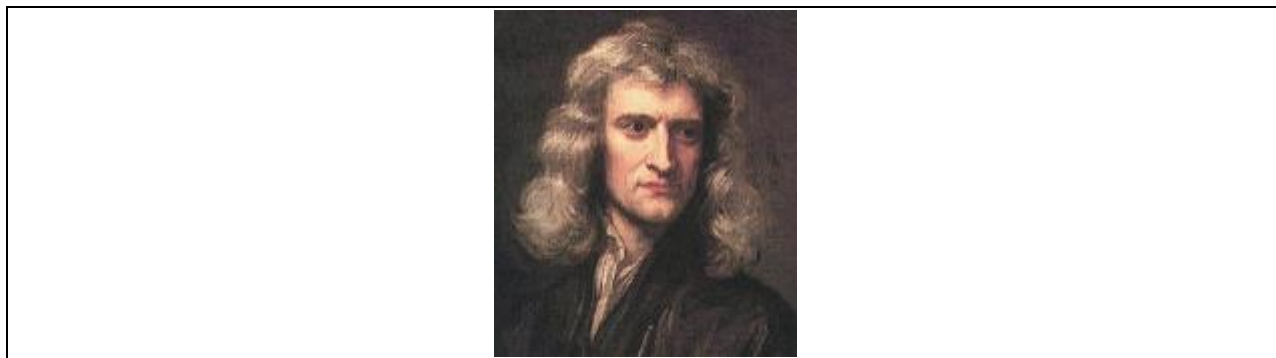
Newton's First Law of Motion

Newton's First Law of Motion

What you have seen is not possible in real life. In reality, it is very difficult to achieve the condition of zero net **force** on the ball. This is because of the presence of the force of friction which acts opposite to the direction of motion of the ball. Thus, in reality, the ball will stop after travelling some distance.

This experiment was first conducted by Galileo Galilei, but the results of his experiment were not widely accepted by the people at that time.

Know Your Scientist



Sir Isaac Newton (1642–1727), the English mathematician, astronomer and physicist, was born at Woolsthorpe. He joined Cambridge University in 1661. He became a fellow of Trinity College in 1667 and Lucasian Professor of Mathematics in 1669. He was at the University till 1696. His famous treatise *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) was prepared during the years 1665–1666. *The Principia*, as it is commonly known, was not published until 1687.

For nearly 300 years, Newton has been considered as the exemplar of modern physical science. His accomplishments in mathematical research are as innovative as those in experimental investigations. He is also known for his works on chemistry, the early history of Western civilisation and theology. Notable among his studies is the investigation of the form and dimensions of the biblical Solomon's Temple.

Isaac Newton used the results of Galileo's experiment to propound the first law of **motion**. It is stated as follows:

A body at rest will remain at rest and a body in uniform motion will continue its uniform motion unless a unbalanced external force acts on it to change its state of rest or uniform motion.

Let us understand Newton's first law of motion with the help of the following examples.



Stationary table

A stationary table remains stationary as long as no one pushes or pulls it. The net force on the stationary table is zero. **An external force is required to change its state of rest.** When someone pushes or pulls the table, it moves along the direction of the applied force.

Pen stand on a table

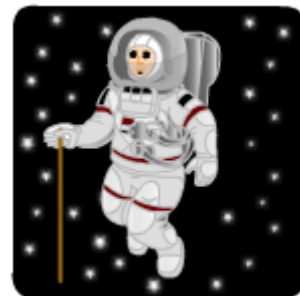


Suppose a pen is lying on a table. The pen stand cannot move by itself, i.e., it cannot change its state of rest by itself. **Its state of rest can be changed only when an external force is applied on it.** For example, if you lift the pen stand from the table, then its state of rest will change. This change is the result of the application of an external force on the pen stand by your hand.

Newton's First Law of Motion

Space walk

An astronaut on a space station goes into outer space for a space walk without any harness. She leaps out from the space station and moves away from it. She has to apply a force toward the space station in order to stop from going too far from it.



What makes the astronaut move away from the space station?

The answer to this question is very simple. There are no forces acting on the astronaut in space. Hence, she keeps moving in a straight line away from the space station. She has to apply a force toward the space station to change her state of uniform motion.

In the context of the preceding examples, you will notice that every body resists a change in its state of motion or rest. If a body is at rest, then it tends to remain at rest; if a body is in a state of motion, then it continues to be in that state of motion. This property of a body is known as **inertia**.

Newton's First Law of Motion

Inertia

Mass is a measure of the amount of inertia

Every object resists changes in its states of motion and rest. This implies that every object has inertia. However, all objects do not have the same tendency to resist changes. This tendency depends upon the mass of an object. Mass is a quantity that is dependent upon the inertia of an object. An object having greater inertia has a greater mass. Hence, a massive object has a greater tendency to resist changes in its states of motion and rest.

Examples of inertia



- When a horse starts running suddenly, the rider falls backward due to the inertia of rest of the upper part of his body.
- The dust particles on a carpet fall off when beaten with a stick. The beating sets the carpet in motion, but the dust particles tend to remain at rest.

Types of inertia

- Inertia of rest
- An object at rest, will remain at rest unless an external force is applied to change its state of rest. For example: on giving a jerk to the branches of a tree, the fruits fall down. This is because on shaking the branches of a tree the fruits attached to it comes in motion. But due to the inertia of rest of the fruits, they tend to remain in this state. Hence, they fall on the ground and attain the state of rest.
- Inertia of motion
- An object in a state of motion will continue to be in the state of motion with the same speed until an external force is applied on it to change its state of motion. For example: When a moving car stops abruptly, the passenger sitting inside it tends to lean forward. This is because when the car is in motion, the whole body of the passenger sitting inside the car is also in motion. On sudden application of brakes, the car and the lower half of the passenger's body (in contact with the car) comes to rest while the upper half remains in motion due to inertia. Thus, the passenger leans forward.
- Inertia of direction
- An object moving in a particular direction will continue to move in that direction until an external force is applied. For example: When a car running on a straight road suddenly takes a right turn, the person inside the car tends to lean leftwards. This is because when the car was moving in a straight road, the whole body of the passenger gained the inertia of moving in straight line. As soon as the car took a right turn, the car and lower half of the person's body changed their direction towards right but the upper half of the person's body still continued to move in a straight line due to inertia of direction. Thus, the person leans leftwards. In the similar way, when the car takes left turn, the person inside the car leans rightwards.

Did You Know?

Inertia always resists a change in the state of motion or rest of a body. Thus, Newton's first law of motion is also known as the law of inertia.

Do all bodies possess inertia?

Yes, all bodies, whether moving or at rest, possess inertia.

Do they possess inertia in the same amount?

We know that pushing a wooden block is easier than pushing an iron block of the same size. We can easily move a football, but it takes a lot of effort to move a large rock. Hence, it



can be said that heavier or massive objects possess greater inertia. Quantitatively, the inertia of an object is measured by its mass.

Momentum

It is a common observation that more force is required to stop a heavier body than what is required for stopping a lighter body. Suppose a cricket ball and table-tennis ball are thrown towards you one after the other. To catch which of the two balls will you need to apply the greater force? The cricket ball, of course! And the reason for this is that it has the greater mass of the two balls. So, we can conclude that the force required to stop the motion of a body is directly proportional to its mass. The same logic is at work when you have to throw the two balls. Since the mass of the cricket ball is greater than that of the table-tennis ball, the force required to throw the former will be greater than that required to throw the latter.

Now suppose you have two cricket balls of the same mass. You throw both the balls, but one with a lesser force than the other. What do you expect will happen? The ball thrown with the greater force will move with a greater velocity as compared to that thrown with the lesser force.

Hence, we can conclude that the effect of force on a body can be described with the help of its mass and velocity. Isaac Newton used the term 'momentum' to describe this effect. He defined momentum as the product of the mass and velocity of a body, i.e.,

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

$$\text{Or, } p = m \times v$$

Where, p = Momentum

m = Mass

v = Velocity

This momentum is also known as linear momentum. You will learn another type of momentum called angular momentum in higher class.

Did You Know?

Force can change the velocity of an object. Thus, force can change the momentum of an object.

Solved Examples



Easy

Example 1: Find the momentum of a cricket ball weighing 150 g and moving at a velocity of 50 m/s.

Solution: It is given that:

Mass of the ball = 150 g = 0.15 kg

Its velocity = 50 m/s

We know that:

Momentum = Mass $\times$ Velocity

$\therefore$ Momentum of the ball = 0.15 kg $\times$ 50 m/s

= 7.5 kg-m/s

Medium

Example 2: A bike weighing 200 kg accelerates from rest at the rate of 5 m/s². Find its momentum after 10 s.

Solution: It is given that:

Initial speed (u) of the car = 0

Its acceleration, $a = 5 \text{ m/s}^2$

Time, $t = 10 \text{ s}$

Let the speed of the car after 10 s be v .

Using the first equation of motion, we can compute the value of v .

$$v = u + at$$

$$\Rightarrow v = 0 + (5 \times 10)$$

$$\Rightarrow \therefore v = 50 \text{ m/s}$$

Now, the mass of the car is given as 200 kg.

So, the momentum of the car after 10 s = Mass $\times$ Velocity

$$= 200 \times 50 = 10000 \text{ kg-m/s}$$

Hard

Example 3: The kinetic energy of a block of mass 3 kg is 150 J. Find its momentum.

Solution: It is given that:

Kinetic energy (k) of the block = 150 J

Its mass, $m = 3 \text{ kg}$

Let the velocity and momentum of the block be v and p respectively.

We know that:

$$k = 0.5 mv^2 \dots (1)$$

$$p = mv \dots (2)$$

Using equation (1), we get:

$$\begin{aligned} k &= \frac{1}{2} mv^2 \\ \Rightarrow k &= \frac{(mv)^2}{2m} \\ \Rightarrow k &= \frac{p^2}{2m} && [\text{From equation (2): } p = mv] \\ \Rightarrow p &= \sqrt{2mk} \\ \Rightarrow p &= \sqrt{2 \times 3 \times 150} \\ \Rightarrow \therefore p &= 30 \text{ kg-m/s} \end{aligned}$$

Newton's Second Law of Motion

Suppose a heavy wooden block is lying on a table. If we give it a gentle push, then it will move with a low velocity. In other words, if we apply a small **force** on the block, then its **momentum** will change slightly. Likewise, if we push the wooden block with a greater force, then the change in its momentum will be greater than before.



We can thus conclude that the change in the momentum of a body is directly proportional to the strength of the applied force. This brings us to Newton's second law of motion.

It is stated as follows:

The rate of change of momentum of an object is directly proportional to the unbalanced external force acting on it. The direction of the unbalanced force is the same as the direction of the change of momentum.

Momentum and Newton's Second Law

Consider a body of mass m . It initially moves with velocity u and accelerates at a constant rate a . It attains a final velocity v after time t . This acceleration is induced by force F .

Now, Newton's second law of motion can be mathematically represented as follows:

The rate of change of momentum $\propto \frac{mv - mu}{t}$

which is equal to unbalanced force, F

$$F \propto \frac{mv - mu}{t}$$

$$F = k \frac{m(v - u)}{t}$$

Where, k is constant of proportionality

Using the first equation of motion, we know that:

$$v = u + at$$

$$\Rightarrow \frac{v - u}{t} = a$$

Using this, we obtain:

$$F = ma = \text{Mass} \times \text{Acceleration}$$

Unit of force is taken Newton so the value of constant of proportionality (k) becomes one.
 $1 \text{ Newton} = 1 \text{ kg } 1 \text{ ms}^{-2}$

Thus, we can restate Newton's second law of motion as follows:

Force acting on a body is equal to the product of its mass and acceleration.

Impulse

Impulse of a force is a measure of the total effect of the force

$$\text{Impulse} = \text{Force} \times \text{Time}$$

Forces which act on bodies for a short time are called impulsive forces.

Example: firing a gun, hitting a ball with a bat

It is a vector quantity.

Solved Examples

Easy

Example 1: A moving block of mass 2 kg changes its speed from 5 m/s to 15 m/s in 2 s. Find the net force acting on the block.

Solution: It is given that:

Initial speed (u) of the block = 5 m/s

Its final speed, $v = 15$ m/s

Time taken, $t = 2$ s

Let the acceleration due to gravity be a .

Using the first equation of motion, we know that:

$$v = u + at$$

$$\Rightarrow a = \frac{v - u}{t}$$

$$\Rightarrow \therefore a = \frac{15 - 5}{2} = 5 \text{ m/s}^2$$

It is given that the mass of the block is 2 kg.



From Newton's second law of motion, we know that:

$$F = ma = 2 \times 5 = 10 \text{ N}$$

Therefore, the net force acting on the block is 10 N.

Medium

Example 2: A particle of mass 2 kg is subjected to a force $F = kx$ with $k = 20 \text{ N/m}$ and x being its distance from the origin. What is its initial acceleration if it is released from a point 30 cm away from the origin?

Solution: It is given that:

Force (F) applied on the particle = kx

Where, $k = 20 \text{ N/m}$

$$x = 30 \text{ cm} = 0.3 \text{ m}$$

$$\therefore F = 20 \times 0.3 = 6 \text{ N}$$

From Newton's second law of motion, we know that:

$$F = ma$$

$$\Rightarrow a = \frac{F}{m}$$

Here, $F = 6 \text{ N}$ and $m = 2 \text{ kg}$.

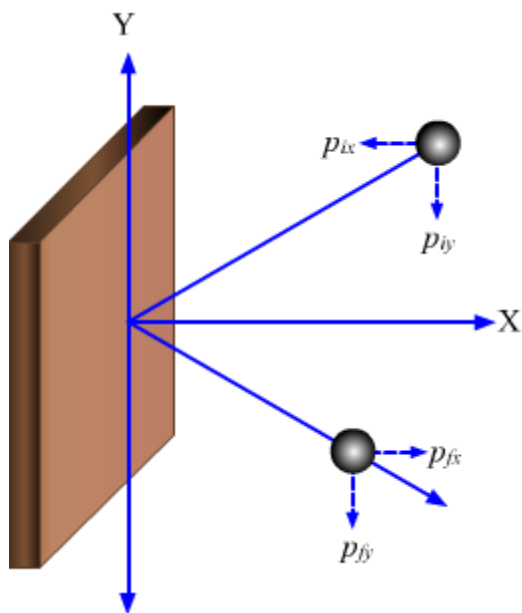
$$\Rightarrow \therefore a = \frac{6}{2} = 3 \text{ m/s}^2$$

Therefore, the initial acceleration of the particle is 3 m/s^2 .

Hard

Example 3: A ball of mass 150 g strikes a wall at a speed of 10 m/s and at an angle of 30° . The ball rebounds with the same speed. If the contact time is 10^{-3} s , then what is the force applied by the wall?

Solution:



Mass (m) of the ball = 150 g = 0.15 kg

Its initial velocity, $u = 10$ m/s

Its initial momentum, $p_i = mu = 0.15 \times 10 = 1.5$ kg-m/s

Initial momentum of the ball along the x -axis, $p_{ix} = -1.5 \cos 30^\circ$

Initial momentum of the ball along the y -axis, $p_{iy} = -1.5 \sin 30^\circ$

Final velocity (v) of the ball = 10 m/s

Its final momentum, $p_f = mv = 0.15 \times 10 = 1.5$ kg-m/s

Final momentum of the ball along the x -axis, $p_{fx} = 1.5 \cos 30^\circ$

Final momentum of the ball along the y -axis, $p_{fy} = -1.5 \sin 30^\circ$

Change in the momentum of the ball along the x -axis = $p_{fx} - p_{ix} = 3 \cos 30^\circ = 3 \times 0.866 = 2.598$ kg-m/s

Change in the momentum of the ball along the y -axis = $p_{fy} - p_{iy} = 0$

So, force acting along the x -axis = $\frac{2.598}{10^{-3}} = 2598$ N

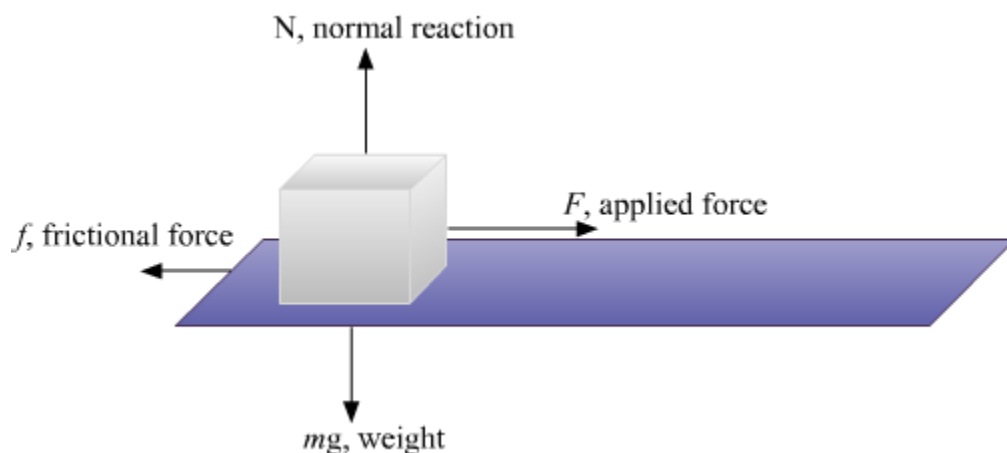
There is no change in the momentum along the y -axis; so, no force acts on the ball along it.

Thus, the force applied by the wall on the ball is 2598 N along the x-axis.

Did You Know?

During the free fall of a ball, the earth pulls the ball toward itself. In turn, the ball also pulls the earth upward with an equal amount of force. However, the effect of this force on the earth is negligible.

Frictional Force



The normal reaction N is equal to the weight mg .

$$N = mg$$

The frictional force f is given by:

$$f = \mu N = \mu mg$$

Where, μ is the coefficient of friction

If the applied force is greater than the frictional force, then the acceleration a of the block is found as:

$$ma = F - f$$

$$\Rightarrow ma = F - \mu mg$$

If the applied force just balances the frictional force, then there is no acceleration of the block and the block does not move.

$$\text{So, } F = f$$

Remember, $a \neq \frac{f}{m}$ when f is the frictional force.

Real-World Examples of Newton's Second Law of Motion

High Jump

During an athletic event, the participants in the high jump event are provided with cushions to fall on after completing a jump. This is done to prevent any kind of injury to the athletes.



When an athlete falls on the cushion, it takes her a longer period of time to come to a stop. A small stopping force acts on her because her rate of change of velocity is low. As a result, she does not get hurt. If the athlete were to fall on a hard surface, then her velocity would reduce to zero in a very short time. In this case, a large stopping force would act on her because her rate of change of velocity would be high. As a result, the athlete would get hurt.

Seat belts



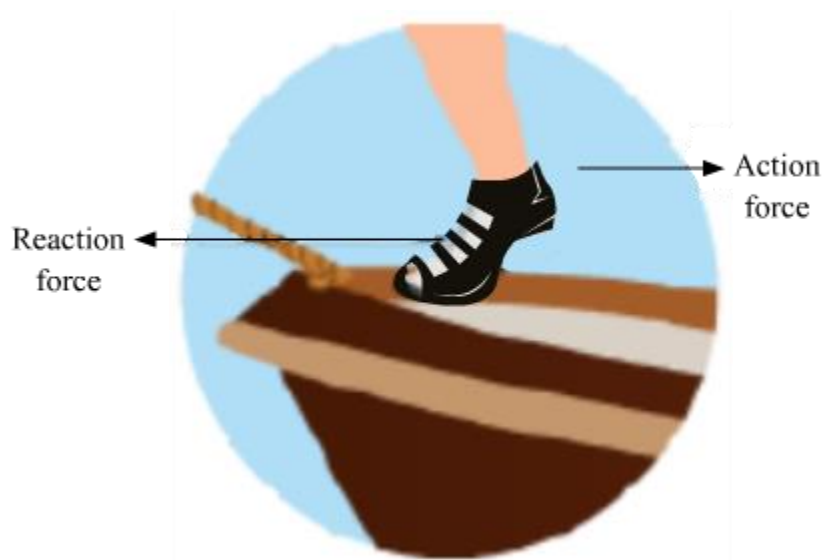
A very useful application of Newton's second law lies in the use of seat belts in cars. To prevent injuries to passengers in case of an accident, all cars are provided with seat belts. In the event of an accident, a fast-moving car stops suddenly, i.e., its high velocity is reduced to zero in a very short interval of time. **The time taken by the passengers to fall**



gets increased because of the seat belts worn by them. The rate of change of velocity of the passengers gets reduced because of the increase in the time taken by them to fall. Hence, a lesser stopping force acts on them, as a result of which, injuries are reduced.

Newton's Third Law of Motion

Newton's third law of motion states that for every action force there is always an equal and opposite reaction force, with the forces acting on different bodies.



the example given in the overview.

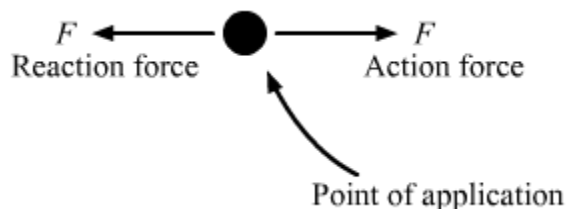
Now, we are going to explain

In order to jump from the boat, Payal applies a force on the boat with her leg. The direction of this force is opposite to that of her motion. As a result, the boat moves backward. In this situation, the applied force is the **action force** and Payal's forward motion is the effect of the **reaction force** provided by the boat. Hence, the boat moves backward because of the action force exerted by Payal.

This situation can be summarized as follows:

Action force → Force exerted by Payal's leg on the boat

Reaction force → Force exerted by the boat on Payal's leg



It is clear from the above example that action and reaction forces are of equal magnitude and act in opposite directions. If Payal applies a force of magnitude F newton on the boat, then the boat in turn reacts with the same magnitude of force on her foot.

It is observed that both balances give the same reading. This implies that the force exerted by balance **II** on balance **I** is the same as the force exerted by balance **I** on balance **II**. Thus,

Action force → Force exerted by balance **II** on balance **I**,

Reaction force → Force exerted by balance **I** on balance **II**

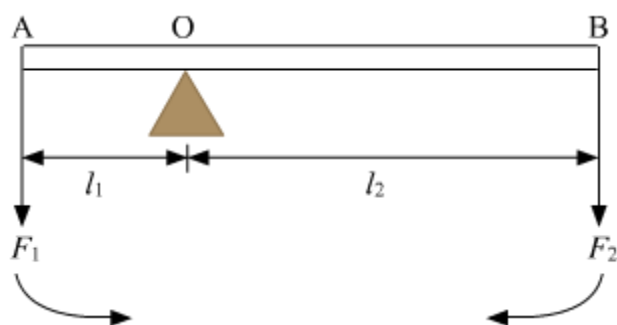
Newton's Third Law of Motion

Rotational equilibrium

Moment is defined as the product of the force and the perpendicular length on a body or system.

A body is in rotational equilibrium when the algebraic sum of moments of all the forces acting on it about a fixed point is zero.

For example: In case of a beam balance or see-saw, the system will be in rotational equilibrium if



$$F_1 \times l_1 - F_2 \times l_2 = 0$$

Now, $F_1 \times l_1 = \tau_1$ (anticlockwise moment)

And, $F_2 \times l_2 = \tau_2$ (clockwise moment)

i.e., for rotational equilibrium, the total external force acting on the body must be zero

Real-Life Applications of Newton's Third Law of Motion

Flying of a bird



A bird can fly with the help of its wings. In this process, it pushes the air downward by flapping its wings. In turn, the air also exerts an equal force on the bottom of its wings. As a result, the bird gets a lift and can fly in the air.

The action–reaction forces in this case are described below.

Action force: Exerted by the wings on the air in the downward direction

Reaction force: Exerted by the air on the bottom of the wings in the upward direction

Horse pulling a cart



The horse can pull and move a cart by exerting a force on the ground. In turn, the horse experiences a reaction force of equal magnitude in the opposite direction that causes the cart to move in that direction. In this case, the **action force** is the force applied by the horse on the ground and the **reaction force** is the force experienced by the horse from the **ground**.

The gun exerts a forward force on the bullet, the bullet in turn also exerts an equal and opposite reaction force on the gun.

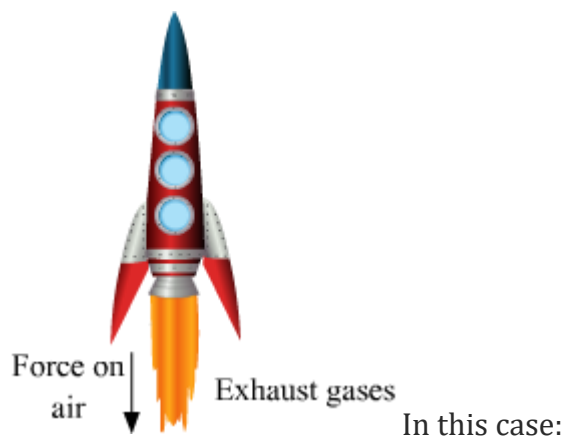
Action force → Force exerted by the gun on the bullet

Reaction force → Force exerted by the bullet on the gun



Rocket

Rockets work on the principle of Newton's third law of motion. In rockets, large amounts of hot gases are allowed to exit through a narrow opening. In turn, the fast-moving gases exert a force on the rocket which pushes the rocket upward.



Action force → Exerted by the rocket on the exhaust gases

Reaction force → Exerted by the gases on the rocket

Solved Examples

Medium

Example: A 600 kg rocket is fired straight up from the earth, with the engines providing 9000 N of thrust. If $g = 10 \text{ m/s}^2$, then the acceleration of the rocket is

1. 5 m/s^2
2. 10 m/s^2
3. 15 m/s^2
4. 50 m/s^2

Solution: It is given that:

Upthrust, $F = 9000 \text{ N}$

Mass (m) of the rocket = 600 kg

$g = 10 \text{ m/s}^2$

Let the acceleration of the rocket be a .

Net force in the upward direction is given as:

Upthrust – Weight of the rocket = Mass × Acceleration

$$\Rightarrow F - mg = ma$$

$$\Rightarrow 9000 - (600 \times 10) = 600a$$

$$\Rightarrow 3000 = 600a$$

$$\therefore a = 5 \text{ m/s}^2$$

Know More

It is a common misconception that rockets are unable to accelerate in space. The fact is that rockets do accelerate in space. They are able to do so because they burn fuel and push the exhaust gases in the direction opposite to the direction in which they need to be accelerated.

Conservation of Linear Momentum

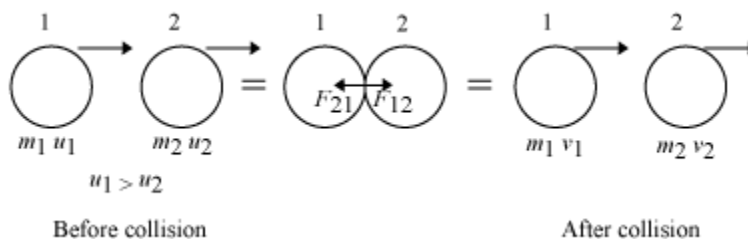
You must have observed a billiard ball being struck by a cue stick. Now, while the rod moves slowly, the ball moves with a very high speed. **Can you relate the velocity of the rod with the velocity of the ball after impact?**

The velocities of two bodies before and after collision can be related to each other with the help of the **law of conservation of momentum**. According to this law:

When two or more bodies act upon one another in a system, the total momentum of the system remains constant, provided there is no external force acting on it.

Conservation of Linear Momentum: In Depth

If a moving ball hits another ball (moving or stationary), then both the balls will have new velocities such that the total momentum of the ball system remains constant. So, the sum of the momentum of each ball before collision is equal to the sum of their momenta after collision (as shown in the figure).



In the shown ball system:

$m_1, m_2 \rightarrow$ Masses of balls 1 and 2 respectively

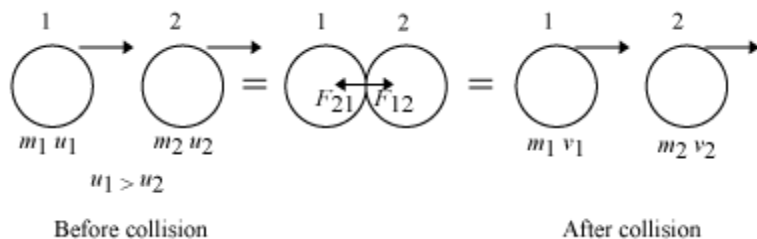
$u_1, u_2 \rightarrow$ Pre-collision velocities of balls 1 and 2 respectively

$v_1, v_2 \rightarrow$ Post-collision velocities of balls 1 and 2 respectively

$F_{12} \rightarrow$ Force exerted by ball 1 on ball 2

$F_{21} \rightarrow$ Force exerted by ball 2 on ball 1

Conservation of Linear Momentum: In Depth



given as:

Before collision: $m_1 u_1 + m_2 u_2$

After collision: $m_1 v_1 + m_2 v_2$

Rate of change of momentum of ball 1 is given as:

$$F_{12} = \frac{m_1 (v_1 - u_1)}{t}$$

Where, $t \rightarrow$ Collision time

Rate of change of momentum of ball 2 is given as:

$$F_{21} = \frac{m_2 (v_2 - u_2)}{t}$$

Using Newton's third law of motion, we can relate the forces F_{12} and F_{21} as:

$$F_{12} = -F_{21}$$

Momentum of the ball system is

$$\Rightarrow \frac{m_1(v_1 - u_1)}{t} = \frac{-m_2(v_2 - u_2)}{t}$$

$$\Rightarrow m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$\therefore$ Momentum of the system before collision = Momentum of the system after collision

The law of conservation of momentum is verified.

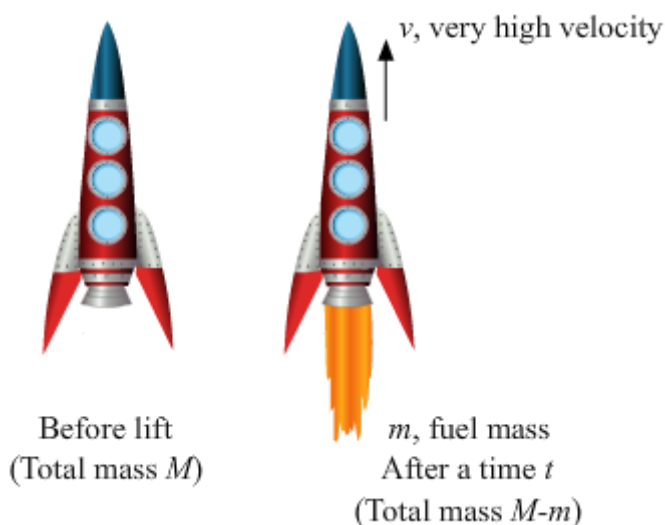
So, in case of the billiard ball and cue stick, the momentum remains constant before and after the striking of the ball by the stick.

Did You Know?

There are four laws of conservation in nature. You have just learnt one of them, i.e., conservation of linear momentum. The other three are:

1. Conservation of mass-energy
2. Conservation of angular momentum
3. Conservation of charge

Application of Conservation of Linear Momentum



Rocket propulsion

The principle of conservation of momentum is used in rocket propulsion. The total momentum of the rocket system before and after propulsion is given as:

Momentum before propulsion: Mass of the rocket + Fuel + Payload before propulsion

Momentum after propulsion: Mass of the rocket + Payload after propulsion

After propulsion, the mass of the system reduces due to the burning of the fuel. This is compensated by the increase in the velocity of the rocket. As a result, the total momentum of the system is kept constant.

Did You Know?

It is a common misconception that rockets are unable to accelerate in space. The fact is that rockets do accelerate in space. They are able to do so because they burn fuel and push the exhaust gases in the direction opposite to the direction in which they need to be accelerated.

Recoil speed of a gun

A gun should be held firmly while firing so that the recoil of the gun does not harm the shoulder. The recoil speed of the gun can be obtained **by applying the law of conservation of momentum**.

Before firing



Let m and M be the masses of the bullet and gun respectively.

Since both are at **rest**, their speeds (u and U) are zero.

Where, u and U is the initial velocity of bullet and gun respectively.

$$\therefore \text{Momentum of the bullet-gun system} = mu + MU = 0 + 0 = 0$$

Hence, there is no momentum before firing.

After firing

Let v and V be the velocities of the bullet and gun respectively.

$\therefore$ Momentum of the bullet–gun system = $mv + MV$

Now, as per the law of conservation of momentum: Momentum before firing = Momentum after firing

$$\Rightarrow 0 = mv + MV$$

$$V = -\left(\frac{m}{M}\right)v$$

The negative sign indicates that the gun recoils in the direction opposite to that of the bullet.

Solved Examples

Easy

Example 1: A gun weighing 5 kg fires a bullet of 25 g with a velocity of 300 m/s. With what velocity does the gun recoil? What is the momentum of the gun and bullet system before and after firing?

Solution: It is given that:

Mass (M) of the gun = 5 kg

Mass (m) of the bullet = 25 g = 0.025 kg

Velocity (v) of bullet = 300 m/s

Let the velocity of the recoil of the gun be V .

Momentum of the gun–bullet system before firing = 0

Momentum of the gun–bullet system after firing = $mv + MV$

Using the law of conservation of momentum, we have

$$mv + MV = 0$$

$$\Rightarrow (0.025 \times 300) + 5V = 0$$

$$\Rightarrow \therefore V = -1.5 \text{ m/s}$$

The negative sign implies that the direction of velocity of the gun is opposite to that of the bullet.

$$\text{Momentum of the gun after firing} = MV = 5 \times (-1.5) = -7.5 \text{ kg-m/s}$$

$$\text{Momentum of the bullet after firing} = mv = 0.025 \times 300 = 7.5 \text{ kg-m/s}$$

Note that the momentums of the gun and bullet are equal and opposite after firing. So, the net momentum of the gun and bullet system is zero.

Before firing, the gun and bullet are at rest; so, the net momentum of the gun and bullet system is zero.

Thus, both before and after firing, the momentum of the gun and bullet system is zero.

Medium

Example 2: With an absolute velocity 5 m/s, a child having a mass 30 kg jumps from a plank of mass 10 kg standing on a smooth surface. Find velocity of the plank and the total energy produced by the child.

Solution: It is given that:

$$\text{Mass } (M) \text{ of the child} = 30 \text{ kg}$$

$$\text{Velocity } (V) \text{ of the child} = 5 \text{ m/s}$$

$$\text{Mass } (m) \text{ of the plank} = 10 \text{ kg}$$

Let the velocity of the plank be v .

$$\text{Initial momentum of the child-plank system} = 0$$

$$\text{Final momentum of the child-plank system} = MV + mv$$

On applying the law of conservation of momentum, we get:

$$MV + mv = 0$$

$$\Rightarrow (30 \times 5) + 10v = 0$$

$$\Rightarrow \therefore v = -15 \text{ m/s}$$

The negative sign shows that the direction of velocity of the plank is opposite to that of the child.

$$\text{Energy produced by the child} = 0.5MV^2$$

$$= 0.5 \times 30 \times 5^2$$

$$= 375 \text{ J}$$

Hard

Example 3: A block of mass 1.0 kg, moving at a speed of 10 m/s, collides with a block of mass 2.0 kg. After collision, the blocks stick together and remain motionless. Calculate the velocity of the 2.0 kg block before collision.

Solution: It is given that:

Mass (m_1) of the first block = 1.0 kg

Its velocity, $u_1 = 10 \text{ m/s}$

Mass (m_2) of the second block = 2.0 kg

Let the velocity of the second block be u_2 .

Initial momentum of the block system = $m_1u_1 + m_2u_2$

Finally, the system comes to rest; so, final momentum of the block system = 0

On applying the law of conservation of momentum, we get

$$m_1u_1 + m_2u_2 = 0$$

$$\Rightarrow (1 \times 10) + 2u_2 = 0$$

$$\Rightarrow \therefore u_2 = -5 \text{ m/s}$$

Before collision, the 2.0 kg block has the velocity 5 m/s in the opposite direction of motion of the 1.0 kg block.

Did You Know?



If an object gets divided into two equal-sized parts as a result of internal forces, then the two parts will fly off in exactly opposite directions.